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Performance Analysis of the Converted Range Rate and Position Linear Kalman Filter

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Abstract—In active sonar and radar applications measurements consist of range, bearing and often range rate - all non-linear functions of the target state (usually modeled in Cartesian coordinates). The converted measurement Kalman filter (CMFK) first converts the range and bearing measurements into Cartesian coordinates to allow for the use of a linear Kalman filter. The extension of the CMKF to use range rate as a linear measurement however has been limited to cases with small bearing errors. The use of range rate as a nonlinear measurement requires the use of a nonlinear filter such as the extended Kalman filter (EKF). Due to the uncertain performance of the EKF, various modifications have been proposed, including use of a pseudo measurement, an alternative linearization of the measurement prediction function, and sequentially processing the converted position and range rate measurements (applied to the EKF and the Unscented Kalman Filter). Common to these approaches is that the measurement prediction function remains nonlinear. A measurement conversion from range, bearing and range rate to Cartesian position and velocity has recently been proposed [4]. This manuscript expands the evaluation of this new approach by comparing to the Sequential EKF, the Sequential Unscented Kalman Filter (UKF) and the posterior Cramer-Rao lower bound (PCRLB). The new method is shown to have improved mean square error performance and exhibits improved constancy over the previously proposed methods, especially in cases with poor bearing accuracy.

I. INTRODUCTION

A common tracking scenario is one in which the measurements consists of range and bearing while the target is tracked in Cartesian coordinates (a natural choice, since target dynamics are linear in the Cartesian coordinate system). A common approach is to employ an EKF to handle the fact that the measurements are a nonlinear function of the target state.

An alternative approach is to first convert the measurements to Cartesian coordinates, allowing the tracking to be performed with a linear Kalman filter. Performance of this approach, referred to as the Converted Measurement Kalman Filter (CMKF), exceeds that of a mixed coordinate EKF if an unbiased conversion from polar to Cartesian coordinates is used [14]. Performance is further enhanced if estimation bias is eliminated by evaluating the converted measurement error covariance using the state prediction [5].

In addition to range and bearing, in many active sonar and radar applications measurements also include range rate.

Previous approaches to incorporate range rate have converted range and bearing to Cartesian coordinates, but left range rate as a nonlinear function of the state. A recently proposed method [4], however, provides a natural extension of the CMKF to include range rate by converting range, bearing and range rate to Cartesian position and velocity. The converted measurement is then used in a linear Kalman filter. The method, referred to here as the converted measurement Kalman filter with range rate (CMKFRR), was shown in [4] to have improved performance over an EKF and an EKF with alternate linearization as describe in [3]. In this manuscript the evaluation is expanded to include the sequential EKF using a pseudo measurement [7], [11], [12] and the sequential UKF using the raw range range measurement [8], [13].

II. PROBLEM STATEMENT

Active sonar and radar systems produce measurements in polar coordinates, often with the additional measurement of range rate:

$$\mathbf{z}_{\text{RAW}} = \begin{bmatrix} r_m \\ \alpha_m \\ \dot{r}_m \end{bmatrix} = \mathbf{h}(\mathbf{x}) \tag{1}$$

where r_m, α_m, and $\dot{r}_m$ are the measured range, bearing and range rate; $\mathbf{h}$ is the measurement function, and $\mathbf{x}$ is the target state. The measurement error for the raw measurements is assumed to be Gaussian with covariance matrix

$$R_{\text{RAW}} = \begin{bmatrix} \sigma_r^2 & 0 & \rho\sigma_r\sigma_{\dot{r}} \\ 0 & \sigma_\alpha^2 & 0 \\ \rho\sigma_r\sigma_{\dot{r}} & 0 & \sigma_{\dot{r}}^2 \end{bmatrix} \tag{2}$$

where σ_r, σ_α, and $\sigma_{\dot{r}}$ are the standard deviations of the range, bearing and range rate measurement noise. The correlation coefficient for the correlation between the range and range rate measurement noise is ρ [10].

Since target motion is linear in Cartesian coordinates, state estimation is best performed in this coordinate system. The Kalman filter for a nearly constant velocity target motion assumption is described in [2]. Defining the state as $\mathbf{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}'$, using H_k and P_k to represent the measurement prediction matrix the state estimate's covariance matrix, the Kalman gain and covariance update steps are

$$S_{k+1} = R_{k+1} + H_{k+1}P_{k+1|k}H'_{k+1} \qquad (3)$$

$$W_{k+1} = P_{k+1|k}H'_{k+1}S^{-1}_{k+1} \qquad (4)$$

$$P_{k+1|k+1} = P_{k+1|k} - W_{k+1}S_{k+1}W'_{k+1} \qquad (5)$$

$$\hat{\mathbf{x}}_{\mathbf{k+1|k+1}} = \hat{\mathbf{x}}_{\mathbf{k+1|k}} + W_{k+1}\left[\mathbf{z_k} - \mathbf{h}\left(\hat{\mathbf{x}}_{\mathbf{k+1|k}}\right)\right] \qquad (6)$$

III. APPROACHES

Two effective techniques that predate the CMKFRR are the sequential EKF using a pseudo measurement and the sequential UKF. Common to these two approaches is that range and bearing are first converted to Cartesian position. This leaves the range rate as the only measurement component that is a nonlinear function of the target state.

Various approaches have been developed for the conversion of range and bearing to Cartesian position. In [14], it was shown that the conventional conversion from polar to Cartesian coordinates introduces a bias in the expected value of the converted measurement. Various remedies to this bias have been proposed [14], [17], [15], [16], [6] and applied to tracking with range rate in [7], [13], [11], [12].

The range rate measurement is a nonlinear function of the target state

$$\dot{r}_m = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} + w_{\dot{r}} \qquad (7)$$

where $w_{\dot{r}}$ is the range rate measurement noise.

The sequential EKF and sequential UKF differ in their approach to handle this nonlinear measurement. The sequential EKF attempts to reduce the nonlinearity between the state and the measurement by replacing the range rate measurement, $\dot{r}_m$, with a pseudo measurement consisting of $r_m\dot{r}_m$.

$$\eta^{\text{pseudo}}_m = r_m\dot{r}_m = h^{\text{pseudo}}_\eta\left(\mathbf{x}\right) + w_\eta = x\dot{x} + y\dot{y} - \rho\sigma_r\sigma_{\dot{r}} + w_\eta \qquad (8)$$

where $\rho\sigma_r\sigma_{\dot{r}}$ is a debiasing term.

The use of this pseudo measurement was proposed in [9] and applied to the second order EKF in [7], [11], [12]. According to [13], the pseudo measurement has disadvantages when the range and range rate measurement noises are not statistically independent (as is the case for certain waveforms [1]). For these cases, use of the UKF has been proposed [8], [13] to handle the strong nonlinearities with the use of range rate instead of the range/range rate product. In this case η_m is simply the range rate measurement, $\dot{r}_m$,

$$\eta^{\text{raw}}_m = \dot{r}_m = h^{\text{raw}}_\eta\left(\mathbf{x}\right) + w_{\dot{r}} = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} + w_{\dot{r}} \qquad (9)$$

The sequential EKF and UKF process the position measurements first, followed by processing of pseudo measurement (8) for the EKF [7], [11], [12] or the raw range range measurement [8], [13] for the UKF. In order to process these measurements sequentially, the range rate based measurement, η^{pseudo}_m or η^{raw}_m, must first be decorrelated from the position components of the measurement. This is achieved as follows.

The covariance matrix of the converted measurement error can be partitioned into the position and pseudo range rate blocks [7]

$$R_{\text{CONV}} = \left[\begin{array}{cc} R^{pp} & R^{p\eta} \\ R^{\eta p} & R^{\eta\eta} \end{array}\right] \qquad (10)$$

where

$$R^{pp} = \left[\begin{array}{cc} R^{xx} & R^{xy} \\ R^{yx} & R^{yy} \end{array}\right], R^{\eta p} = \left[\begin{array}{cc} R^{x\eta} & R^{y\eta} \end{array}\right] \qquad (11)$$

Let

$$L = -R^{\eta p}\left(R^{pp}\right)^{-1} = \left[\begin{array}{cc} L_1 & L_2 \end{array}\right] \qquad (12)$$

and

$$B = \left[\begin{array}{cc} I_{2x2} & 0 \\ L & 1 \end{array}\right] \qquad (13)$$

By pre-multiplying B on both sides of the measurement conversion equation, a new measurement prediction function can be obtained in which the position measurement is unmodified, and the pseudo range rate is replaced with a decorrelated pseudo range rate, ε

$$\varepsilon \triangleq L_1x_m + L_2y_m + \eta_m \qquad (14)$$

with the corresponding measurement prediction function

$$h^{\text{pseudo}}_\varepsilon = L_1x + L_2y + x\dot{x} + y\dot{y} \qquad (15)$$

for the pseudo range rate approach, and

$$h^{\text{raw}}_\varepsilon = L_1x + L_2y + \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}} \qquad (16)$$

for the raw range rate approach.

For the second order EKF with decorrelated pseudo range rate measurements, (3) – (6) are processed for the converted position portion of the measurements. The state estimate, $\hat{\mathbf{x}}^{\text{P}} = \left[\begin{array}{cccc} \hat{x} & \hat{y} & \hat{\dot{x}} & \hat{\dot{y}} \end{array}\right]'$, and state covariance, $P^{\text{p}}_{k+1|k+1}$, updated using the position measurement only, are subsequently processed using the decorrelated pseudo range rate measurement in a second order EKF [7].

Similarly, in the sequential UKF, (3) – (6) are processed for the converted position portion of the measurements. The state estimate, $\hat{\mathbf{x}}^{\text{P}}$, and state covariance estimate, $P^{\text{p}}_{k+1|k+1}$, updated using the position measurement only, are subsequently processed by the decorrelated range rate measurement using the second order unscented transform [13]. Sigma points are generated using the state and covariance estimate that has been updated by the position estimates and then passed through the nonlinear function, $h^{\text{raw}}_\varepsilon$, to provide the time and measurement update.

IV. NEW CONVERTED MEASUREMENT APPROACH WITH RANGE RATE MEASUREMENTS

The approach used in the recently introduced converted measurement Kalman filter with range rate (CMKFRR) is to convert the raw measurement of range, bearing and range rate into a measurement of position and velocity in Cartesian coordinates. The raw measurement is converted in a manner that

is unbiased and consistent, and that allows for and describes the correlation between the range and range rate measurement errors. The converted measurement error covariance estimate is evaluated at the prediction (as opposed to the measurement) to avoid estimation bias [5].

In order to develop this conversion, consider the inclusion of a non-informative cross range rate measurement, $\dot{c}$. The conversion function to Cartesian is therefore,

$$\mathbf{z}_{\mathrm{C}} = \begin{bmatrix} x_m \\ y_m \\ \dot{x}_m \\ \dot{y}_m \end{bmatrix} = \mathbf{D}\left(\alpha_{\mathbf{m}}\right) \begin{bmatrix} r_m \\ 0 \\ \dot{r}_m \\ \dot{c}_m \end{bmatrix} \quad (17)$$

where D is the direction cosine matrix,

$$\begin{bmatrix} \cos\alpha_m & -\sin\alpha_m & 0 & 0 \\ \sin\alpha_m & \cos\alpha_m & 0 & 0 \\ 0 & 0 & \cos\alpha_m & -\sin\alpha_m \\ 0 & 0 & \sin\alpha_m & \cos\alpha_m \end{bmatrix} \quad (18)$$

Cross range rate is non-informative because it is not truly measured, but a priori knowledge about the distribution of expected cross range rates (based on knowledge of possible target speeds) can be used in calculating the converted measurement error covariance as describe in IV-B.

A. Estimation of the Mean

The conversion (17) has multiplicative bias of a factor of $e^{-\sigma_\alpha/2}$. An unbiased version of the measurement conversion (17) can be developed as an extension of the Unbiased Converted Measurement for position [15]

$$\mathbf{z}_{\mathrm{U}} = \begin{bmatrix} x_m \\ y_m \\ \dot{x}_m \\ \dot{y}_m \end{bmatrix} = e^{\sigma_\alpha^2/2} D\left(\alpha_m\right) \begin{bmatrix} r_m \\ 0 \\ \dot{r}_m \\ \dot{c}_m \end{bmatrix} \quad (19)$$

B. Estimation of the Covariance

For convenience, the converted measurement error covariance, R_{C}, will be developed in a coordinate system along the line of sight (LOS) to the target, R_{R}, and then converted to Cartesian coordinates, i.e.

$$R_{\mathrm{C}} = D\left(\alpha\right) R_{\mathrm{R}} D\left(\alpha\right)' \quad (20)$$

The calculation of the components of R_{C} requires the true target velocity and position. Since these are not available in practice, the evaluation is performed at the predicted target state, $\hat{\mathbf{x}}_{k+1|k}$.

First the predicted target state and covariance are rotated into the estimate's LOS coordinate system:

$$\hat{\mathbf{x}}_{\mathrm{R}} = D\left(\alpha_t\right)' \hat{\mathbf{x}}_{k+1|k} \quad (21)$$
$$P_{\mathrm{R}} = D\left(\alpha_t\right)' P_{k+1|k} D\left(\alpha_t\right) \quad (22)$$

where the predicted target bearing is

$$\alpha_t = \tan^{-1}\left(\frac{\hat{\mathbf{x}}_{k+1|k}^2}{\hat{\mathbf{x}}_{k+1|k}^1}\right) \quad (23)$$

and $\hat{\mathbf{x}}^n$ is the nth component of $\hat{\mathbf{x}}$.

The individual components of R_{R} evaluated at the prediction are as follows,

$$R_{\mathrm{R}}^{11} = \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^1\right)^2 + P_{\mathrm{R}}^{11} + \sigma_r^2\right)\beta_+ - \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^1\right)^2 + P_{\mathrm{R}}^{11}\right)\delta_+ \quad (24)$$

$$R_{\mathrm{R}}^{12} = 0 \quad (25)$$

$$R_{\mathrm{R}}^{13} = \left(\hat{\mathbf{x}}_{\mathrm{R}}^1 \hat{\mathbf{x}}_{\mathrm{R}}^3 + P_{\mathrm{R}}^{13} + \rho\sigma_r\sigma_{\dot{r}}\right)\beta_+ - \left(\hat{\mathbf{x}}_{\mathrm{R}}^1 \hat{\mathbf{x}}_{\mathrm{R}}^3 + P_{\mathrm{R}}^{13}\right)\delta_+ \quad (26)$$

$$R_{\mathrm{R}}^{22} = \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^1\right)^2 + P_{\mathrm{R}}^{11} + \sigma_r^2\right)\beta_- - \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^1\right)^2 + P_{\mathrm{R}}^{11}\right)\delta_- \quad (27)$$

$$R_{\mathrm{R}}^{23} = \left(\hat{\mathbf{x}}_{\mathrm{R}}^1 \hat{\mathbf{x}}_{\mathrm{R}}^4 + P_{\mathrm{R}}^{14}\right)\beta_- - \left(\hat{\mathbf{x}}_{\mathrm{R}}^1 \hat{\mathbf{x}}_{\mathrm{R}}^4 + P_{\mathrm{R}}^{14}\right)\delta_- \quad (28)$$

$$R_{\mathrm{R}}^{33} = \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^3\right)^2 + P_{\mathrm{R}}^{33} + \sigma_{\dot{r}}^2\right)\beta_+ + \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^4\right)^2 + P_{\mathrm{R}}^{44} + \sigma_{\dot{c}}^2\right)\beta_-$$
$$- \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^3\right)^2 + P_{\mathrm{R}}^{33}\right)\delta_+ - \left(\left(\hat{\mathbf{x}}_{\mathrm{R}}^4\right)^2 + P_{\mathrm{R}}^{44}\right)\delta_- \quad (29)$$

where

$$\beta_\pm = \frac{1}{2}\left[1 \pm e^{-2\sigma_\alpha^2} e^{-2\sigma_{\alpha_t}^2}\right]e^{\sigma_\alpha^2} \quad \delta_\pm = \frac{1}{2}\left[1 \pm e^{-2\sigma_{\alpha_t}^2}\right]$$

and $\sigma_{\alpha_t}^2$ is the approximate bearing variance of the predicted track estimate based on a linearization of P_{R},

$$\sigma_{\alpha_t}^2 = \frac{P_{\mathrm{R}}^{22}}{\left(\hat{\mathbf{x}}_{\mathrm{R}}^1\right)^2} \quad (30)$$

$\hat{\mathbf{x}}_{\mathrm{R}}^n$ is the nth component of $\hat{\mathbf{x}}_{\mathrm{R}}$ and P_{R}^{nm} is the (nm) element of P_{R}. The value of $\sigma_{\dot{c}}$ used in (29) is set based on an a priori estimate of the standard deviation of target cross range rate.

Since the cross range rate measurement, $\dot{c}_m$, is non-informative, the remaining components of the measurement noise covariance in the LOS coordinate system, R_{R} (e.g. R_{R}^{44}, R_{R}^{34}) are infinite. It is therefore useful to deal with the inverse of R_{R}

$$R_{\mathrm{R}}^{-1} = \begin{bmatrix} \left(R_{\mathrm{R}}^{1:3,1:3}\right)^{-1} & \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \\ 0 \quad 0 \quad 0 & 0 \end{bmatrix} \quad (31)$$

Since the inverse of the direction cosine matrix, $D\left(\alpha_m\right)$, is its transpose, the measurement noise covariance for (19), R_{C}, is

$$R_{\mathrm{C}}^{-1} = D\left(\alpha_t\right) R_{\mathrm{R}}^{-1} D\left(\alpha_t\right)' \quad (32)$$

The inconsistency in the use of $\sigma_{\dot{c}}$, and the rational, requires explanation. The value in (29) influences the Kalman gain for the $\dot{r}_m$ measurement and an a prior estimate of $\sigma_{\dot{c}}$ is used to improve the consistency of the tracker. For the components of R_{R} that influence the Kalman gain for $\dot{c}_m$, $\sigma_{\dot{c}}$ is set to infinity (or, equivalently, the components of R_{R}^{-1} set to zero). This is to ensure that the tracker is not biased towards the value substituted for $\dot{c}_m$, since it is not truly measured.

The consistency of the conversion method was examined using the Normalized Error Squared (NES) [2] and shown to be consistent in [4]. Note that although the converted measurement has dimension 4, the expected NES [2, eq. (10.4.3-26)] is 3, since velocity errors along the cross range are multiplied by zero.

Unfortunately, R_C^{-1} is not invertible, so R_C is not available for use directly in the Kalman filter gain calculation (3). It can however be used in the information form of the Kalman filter, described in V-A.

V. Application to Tracking

A. Information Form of the Kalman Filter

The information form of the Kalman filter [2] propagates the inverse of the state covariance and uses the inverse of the measurement error covariance. This allows the use of R_C^{-1} (32) directly, in place of R_C, which is unavailable. The calculation of the Kalman gain in the Kalman filter (3) – (4) is replaced with

$$W_{k+1} = \left[P_{k+1|k}^{-1} + H'R_{k+1}^{-1}H\right]^{-1} H'R_{k+1}^{-1} \qquad (33)$$

and the state covariance update (5) is replaced with

$$P_{k+1|k+1} = [P_{k+1|k}^{-1} + H'R_{k+1}^{-1}H]^{-1} \qquad (34)$$

B. Converted Measurement Kalman Filter with Range Rate

With the use of the measurement conversion function (19), each component of the state (for the nearly constant velocity model) is observed directly. When applied to the information form Kalman filter, one has H as the identity matrix. The converted measurement is therefore linear with respect to the target state, eliminating the need for the extended (or unscented) Kalman filter.

The Converted Measurement Kalman Filter with Range Rate (CMKFRR) is implemented as follows:

1) Convert the raw measurements of range, bearing and range rate to Cartesian position and velocity with (19), using $\dot{c}_m = 0$, and use the result, $\mathbf{z} = \mathbf{z}_\text{U}$, in (6).
2) Use the information form of the Kalman filter (33)–(34) with $R^{-1} = R_\text{C}^{-1}$ from (32).
3) Set the measurement prediction matrix, $H = I_{4\times4}$, in (33)–(34).
4) Let $h(\hat{\mathbf{x}}) = \hat{\mathbf{x}}$ in (6).

VI. Evaluation

A. Performance Comparisons

The performance of the proposed Converted Measurement Kalman Filter with Range Rate (CMKFRR) has been evaluated with respect to the current state-of-art techniques. The CMKFRR was compared to

1) CMKF using range and bearing measurements only (POS)
2) Sequential EKF using pseudo range rate (i.e. range, range rate product) as described in III (SEKF)
3) Sequential UKF using range-rate as described in III (SUKF)
4) Posterior Cramer-Rao lower bound, as defined in [18], for range and bearing measurements only (PCRLBPOS)
5) Posterior Cramer-Rao lower bound, as defined in [18], for range, bearing and range-rate measurements (PCRLBPOSRR)

To allow for direct comparison, all of the existing trackers were implemented with the conversion of range and bearing to Cartesian coordinates using the method described in [6], [12]. The conclusions hold for other conversion methods.

Measures of performance include mean square error (MSE) for the target position and velocity estimates. An additional measure of performance is the tracker consistency based on the Average Normalized Estimation Error Squared (ANEES) [2]. The ANEES of a consistent estimator is close to 1. Tracker consistency is important not only for analysis of results, but also for measurement to track association in multitarget tracking scenarios in clutter.

The tracking scenario is set up as follows:

1) True target range, r, normally distributed with mean 4,000m and standard deviation of 30m.
2) True target bearing uniformly distributed from $-\pi$ to π
3) True target heading uniformly distributed from $-\pi$ to π
4) True target speed χ_2 distributed, scaled by 10m/s
5) Sensor range accuracy, σ_r = 30m
6) Sensor bearing accuracy, σ_α = 1°, 2.5°, 5°, 8° and 16°.
7) Sensor range rate accuracy, $\sigma_{\dot{r}}$ = 0.1m/s
8) Correlation between range and range rate errors, $\rho = -0.2$.
9) $\sigma_{\dot{c}}$ in (29) set to 10 m/s

Fig. 1 shows the results of 5,000 Monte Carlo runs of the trackers. In each of the test cases, the proposed CMKFRR achieved performance that was equal to or better than the existing methods. For small angle error, the performance of the CMKFRR matched the existing methods (results overlap on plots), and all trackers were fairly consistent. As the angle accuracy degraded, the CMKFRR performance was better in terms of MSE, and considerably better in terms of consistency. Even for severely degraded angle accuracy, σ_α = 16°, the CMKFRR maintained consistency.

VII. Conclusion

When tracking with measurements of range, bearing and range rate, various filtering techniques have been proposed. For cases with poor bearing accuracy, the valid approaches have been limited to nonlinear filters, such as the EKF and UKF. This work has shown that a linear Kalman filter can be employed to handle this estimation problem by using a new converted measurement technique and the information form of the Kalman filter. The conversion process avoids conversion bias by using an unbiased converted measurement, and precludes estimation bias by evaluating the converted measurement error covariance at the prediction. Simulations show that this linear Kalman filter has improved MSE performance over all the state of the art trackers, including the sequential EKF using pseudo range rate and the sequential UKF. This new approach is recommended for consideration in active radar or sonar systems that include measurements of range rate.
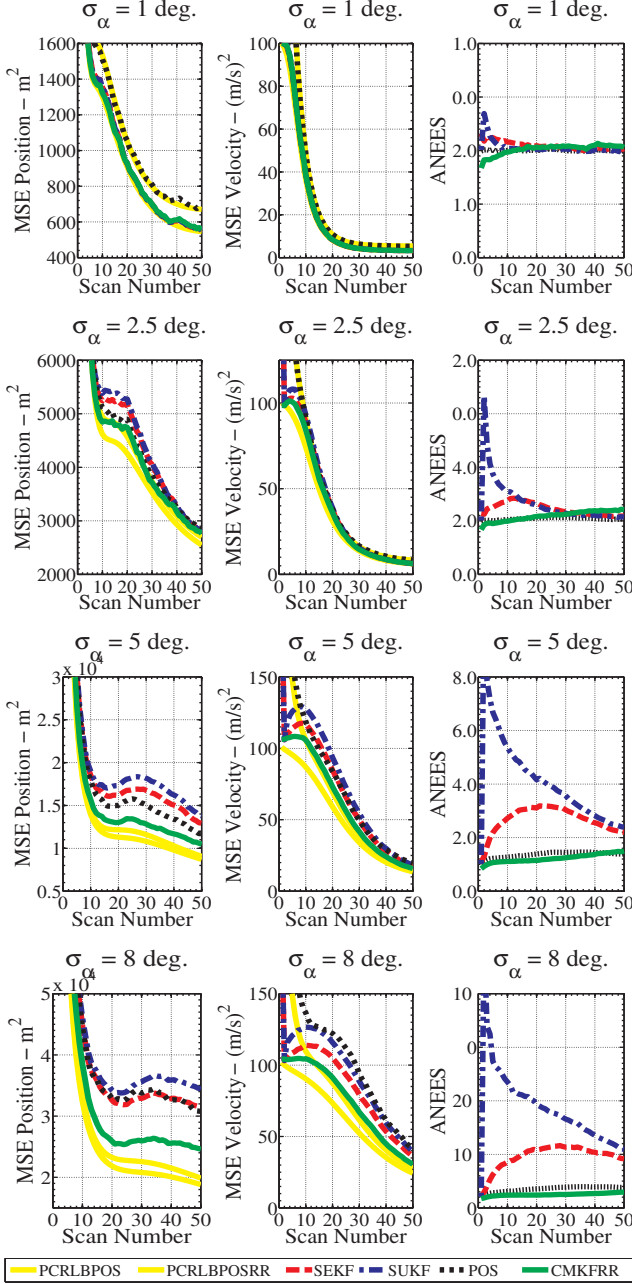
Fig. 1. MSE and ANEES for the position only based tracker (POS), sequential EKF using pseudo range rate (SEKF), sequential UKF using range-rate (SUKF), and the new Converted Measurement Kalman Filter with Range Rate (CMKFRR). The PCRLB for position only measurements and position measurements with range rate is also shown.

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